LA-UR--85-1793

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SUBMITTED TO: Seventh U.S. Symposium on Compact Toroid Research, Santa Fe, NM, May 21-23, 1985

FRC TRANSLATION INTO A COMPRESSION COIL

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I. Introduction: The equilibrium and translational kinematics of Field-Reversed Configurations (FRCs) in a cylindrical coil which does not conserve flux are problems that arise in connection with adiabatic compressional heating. The usual treatments of the equilibrium problem for FRCs specify a fixed amount of open flux within a conducting boundary,[1,2] and in previous work the kinematics of translation between two flux conservers have been discussed.[3,4] However, in order to most efficiently heat the FRC plasma, it is desirable to translate the FRC, after it is formed, into a close-fitting compression coil and then to adiabatically compress the plasma by increasing the open flux.[5] The compression coil, or "flux shaper", must be a flux surface on the time scales for translation (τ_{trans}) and compression (τ_{comp}) in order to maintain the axial equilibrium of the FRC, but the amount of open flux is influenced by the external circuit. An additional problem is that, unless the energy confinement time τ_E is so long that $\tau_{trans} << \tau_{comp} << \tau_E$ can be satisfied, a separate guide field in addition to the compression field is needed in the compression coil to confine the FRC prior to compression. Coupling between the guide field and compression field systems must then be considered.

In this paper, we consider several features of the problem of FRC translation into a compression coil. First, the magnitude of the guide field is calculated and found to exceed that which would be applied to a flux conserver. Second, energy conservation is applied to FRC translation from a flux conserver into a compression coil. It is found that a significant temperature decrease is required for translation to be energetically possible. The temperature change depends on the external inductance in the compression circuit. An analogous case is that of a compression region composed of a compound magnet; in this case the temperature change depends on the ratio of inner and outer coil radii. Finally, the kinematics of intermediate translation states are calculated using an "abrupt transition" model. It is found, in this model, that the FRC must overcome a potential hill during translation, which requires a small initial velocity.

II. Translation into a Compression Coil: We consider the situation in which a guide field is present in the compression coil to confine the translating FRC prior to compression. The guide field can be applied either by switching a capacitor bank C_B into the compression coil of inductance L_{co} through an isolating inductor L_B (Fig. 1) or by energizing a separate coil outside the compression coil. Considering the former case first, we assume that the guide field bank is crowbarred at peak current to provide the guide field. By equating the flux in the circuit before and after the FRC enters the compression coil ($(L_B + L_{co})I_o = (L_B + L_c)I$), we find (neglecting finite length corrections) that

$$B_o = (1 - x_s^2)B_w \frac{1 + L_B/L_c}{1 + L_B/L_{co}}. \qquad (1)$$

In this expression B_o is the guide field, B_w is the confining field at the wall after the FRC has entered, x_s is the ratio of FRC separatrix and coil radii, and $L_c < L_{co}$ is the coil inductance in the presence of the FRC.

Using long-coil and cylindrical FRC assumptions for L_c ($L_c = L_{co}(1 - x_s^2)/(z_s + (1 - x_s^2)(1 - z_s))$ where z_s is the ratio of FRC length ℓ_s to coil length ℓ_c) the guide field can be written as

$$B_o = B_w(1 - x_s^2 + f_L z_s x_s^2) \tag{2}$$

where $f_L \equiv L_B/(L_B + L_{co})$. The guide field required in the compression coil for given x_s and B_w is larger than that needed in a flux conserver where $L_B = 0$ and can approach B_w if $L_B >> L_{co}$ and the FRC fills the length of the coil. The external inductance has the effect of reducing the increase in total current when the FRC enters, so more current is needed initially.

The axial kinetic energy of the FRC can be determined by considering the total energy of the system consisting of FRC, formation coil, and compression coil. The formation coil is assumed to be crowbarred and acting as a flux conserver. Before translation, the total energy is[3,4]

$$\frac{5}{2}NT_1 + U_{1,vac} + \frac{1}{2}Nm_i v_1^2 + U_{2,vac} \tag{3}$$

where N and T_1 are the particle inventory and temperature in the formation region, $U_{1,vac}$ and $U_{2,vac}$ are the magnetic energies in the formation and compression circuits in the absence of plasma, m_i is the ion mass, and v_1 is the initial axial velocity. After translation, the total energy is

$$U_{1,vac} + \frac{3}{2}NT_2 + \frac{1}{2}Nm_i v_2^2 + U_2 \tag{4}$$

where T_2 and v_2 are the temperature and velocity following translation, U_2 is the magnetic energy in the compression circuit and the FRC, and where losses are ignored.

The change of magnetic energy in the compression region, $\Delta U_2 \equiv U_2 - U_{2,vac}$, can be written as

$$\Delta U_2 = \frac{1}{2}L_B(I^2 - I_o^2) + (\frac{1}{2}L_c I^2 - \frac{1}{2}L_{co}I_o^2) + V_{FRC}(1 - \langle\beta\rangle)\frac{B_w^2}{8\pi} \tag{5}$$

where $\langle\beta\rangle$ is the volume-averaged beta within the separatrix. The first two terms represent the energy changes in the external inductor and the coil, respectively, and the third term is the magnetic energy within the separatrix. After some algebra, this can be written as

$$\Delta U_2 = NT_2(1 + \frac{z_s x_s^2 f_L}{\langle\beta\rangle}) \tag{6}$$

In deriving this expression, Eq. 2 and the approximate expression for coil inductance were used.

By equating (3) and (4) and using (6), we find that

$$T_2 = \frac{T_1 + \frac{1}{5}m_i(v_1^2 - v_2^2)}{1 + \frac{2}{5}\frac{z_s x_s^2 f_L}{\langle\beta\rangle}}. \tag{7}$$

This equation reduces to the result for translation between flux conservers[3,4] when $f_L = 0$ and shows that T_2 is lower when $f_L > 0$. The magnitude of this decrease can be important. In the case of the proposed FRX-D experiment,[6] $x_s = 0.8$, $f_L \approx 0.7$, $z_s \approx 1.0$, and $\langle\beta\rangle = 0.68$ so that T_2 is reduced by 21%. Thus either the formation region must be designed to form the FRC at higher temperature to yield the desired temperature before compression, or the compression bank must be increased to reach the desired final temperature.

We now consider the case in which the compression region is a compound magnet. The guide field is then provided by dc magnet coils located outside the compression coil. In general, the guide field coils must be protected from the voltages induced by the translation and compression of the FRC. Thus their protective cladding is a flux conserver on the translation and compression time scales. The compression coil is a flux surface on these time scales and carries the azimuthal currents needed to maintain the axial equilibrium of the FRC. However, the net azimuthal current must be zero since the compressional coil circuit is open before compression. These constraints result in the relation between the guide and confining fields

$$B_o = B_w[1 - x_s^2 + (1 - y^2)z_s x_s^2] \qquad (8)$$

where y is the ratio of flux shaper and flux conserver radii. This equation is similar to (2) except that $(1 - y^2)$ replaces f_L. A calculation of the total magnetic energy in the compound magnet compression region yields a result which differs from (6) in the same way. Thus a lower final temperature is necessary in either case. Of course, either $(1 - y^2)$ or f_L must be kept close to unity to prevent excessive coupling between the compression flux and coil cladding or between the compression bank and guide field inductor, respectively.

III. Kinematics of the Transition: We have seen that translation into a compression coil requires a larger guide field than translation into a flux conserver. However, when only a fraction of the FRC length has entered the compression coil, Eqs. (2) and (8) show that it encounters a larger confining field B_w than when its full final length is within the coil. Thus the FRC may encounter a potential hill between the initial and final states. In practice, this effect may be mitigated by a transient modification of the FRC equilibrium or by designing a tapered compression coil to soften the transition in radius. In order to estimate the magnitude of this effect, we have calculated the total energy of the intermediate states using an "abrupt transition" model which may provide an upper bound on the size of the potential hill.

In this model, we assume that a cylindrical FRC equilibrium encounters an abrupt change in wall radius from the formation coil to the compression coil. We further assume that the response of the portion of the FRC which has entered the compression coil is governed by the adiabatic laws[5] for FRC equilibria and by Eq. (2). These adiabatic laws were found to be in reasonable agreement with FRC translation experiments[4] in which FRCs traversed a change in wall radius of 0.74 at axial speeds approaching the ion thermal speed; thus it may be reasonable to use them in this model.

The internal flux of the FRC, assuming a typical diffuse profile, is given by $\phi = r_w^2 B_w x_s^{3.25}$. Applying conservation of flux across the transition, we have

$$(1 - x_{s2}^2 + f_L z_2 x_{s2}^2)\left(\frac{B_{w1}}{B_o}\right)\left(\frac{r_{w1}}{r_{w2}}\right)^2\left(\frac{x_{s1}}{x_{s2}}\right)^{3.25} = 1. \qquad (9)$$

The FRC length ℓ_2 within the compression coil is estimated using the adiabatic laws for the portion $\nu\ell_1$ of the initial FRC length which crossed the transition:

$$\ell_2 = z_2\ell_c = \nu\ell_1\left(\frac{r_{w2}}{r_{w1}}\right)^{0.4}\left(\frac{x_{s2}}{x_{s1}}\right)^{1.9}\left(\frac{\langle\beta_2\rangle}{\langle\beta_1\rangle}\right)^{-0.7}. \qquad (10)$$

Equations 10 and 11 can be numerically solved for x_{s2} as a function of ν. This permits the total energy of the intermediate state to be calculated as a function of ν using the adiabatic law to estimate T_2:

$$T_2 = T_1\left(\frac{r_{w2}}{r_{w1}}\right)^{-1.6}\left(\frac{x_{s2}}{x_{s1}}\right)^{-2.6}\left(\frac{\langle\beta_2\rangle}{\langle\beta_1\rangle}\right)^{0.3}. \qquad (11)$$

Comparison of the total energies of the initial and intermediate states yields

$$\frac{1}{2}m_i v_1^2 + \frac{5}{2}T_1 = \frac{1}{2}m_i v_2^2 + \frac{5}{2}(1 - \nu)T_1 + \frac{5}{2}\nu T_2 + \nu T_2\frac{z_2 x_{s2}^2 f_L}{\langle\beta\rangle}. \qquad (12)$$

The results of this calculation for the parameters of FRX-D are illustrated in Fig. 2. A potential hill is observed with a magnitude of about 18 eV/particle. This requires an initial velocity of about 4 cm/μs to overcome, which is relatively easy to produce in present FRC translation experiments.[4] Thus this effect does not appear to be a serious obstacle to a translation and compression experiment.

IV. Conclusions: FRC translation into a compression coil in an adiabatic compressional heating experiment differs significantly from translation between flux conservers. It is shown that translation into a compression coil requires more guide field and a lower final temperature because of the interaction between the compression circuit and the two possible methods by which guide field can be applied. These effects are of sufficient magnitude to influence the design of a compressional heating experiment. An additional potential hill may be present between the initial and final states, but it can be overcome by a modest initial translation velocity.

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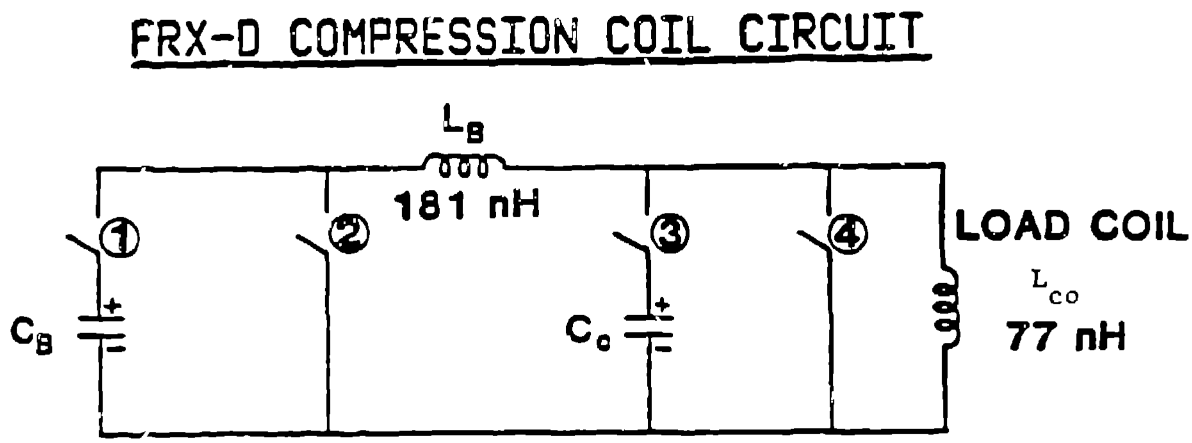

Figure 1. Compression coil circuit for FRX-D, showing the guide field (C_B) and compression field (C_C) capacitor banks and the isolation (L_B) and load coil (L_{co}) inductances.

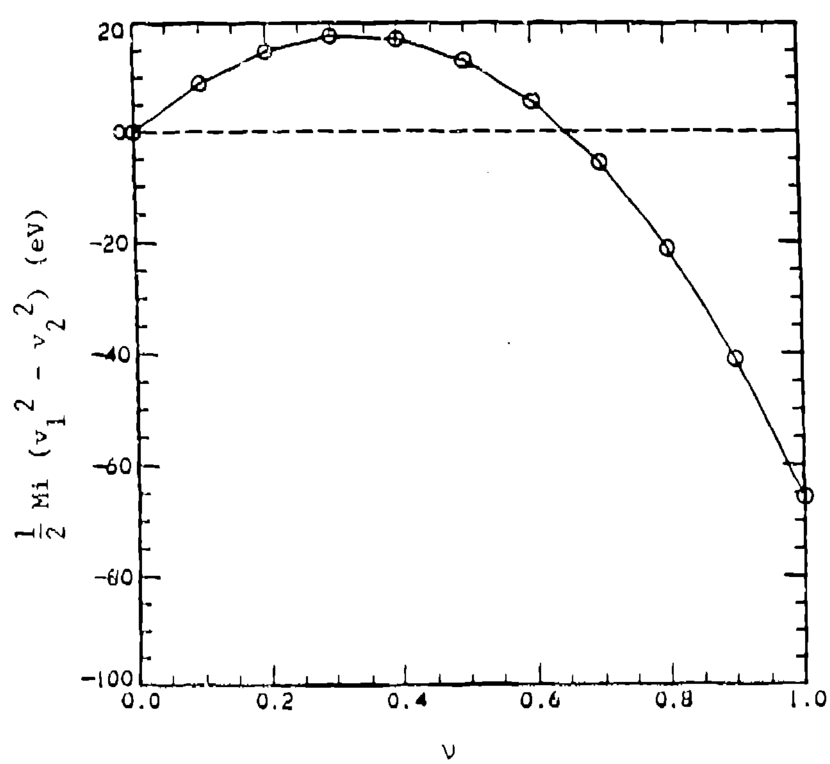

Figure 2. Prediction of the "abrupt transition" model for the translational potential energy per ion as a function of the fraction ν of the FRC which has crossed the transition.